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Sizable CP Violation in the Bosonized Standard Model

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Using the worldline method, we derive an effective action of the bosonic sector of the Standard Model by integrating out the fermionic degrees of freedom. The CP violation stemming from the complex phase in the CKM matrix gives rise to CP-violating operators in the one-loop effective action in the next-to-leading order of a gradient expansion. We calculate the prefactor of the appropriate operators and give general estimates of CP violation in the bosonic sector of the Standard Model. In particular, we show that the effective CP violation for weak gauge fields is not suppressed by the Yukawa couplings of the light quarks and is much larger than the bound given by the Jarlskog determinant.

I. INTRODUCTION

It is a well established fact that CP violation in the Standard Model is very small. Main reason for this is that the sole CP-violating effects stem from the Yukawa couplings of the quarks. In particular, the Yukawa sector is constrained by the special flavor structure of the Standard Model [1, 2] suppressing CP violation. To be explicit, the CP violation arises due

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to the following terms in the Lagrangian

$$Y^u_{ij}\,\bar{Q}^i_L\,u^j_R\phi + Y^d_{ij}\,\bar{Q}^i_L\,d^j_R\tilde{\phi} + h.c., \tag{1}$$

where Q_L denotes the left-handed quark $SU(2)_L$ doublet, d_R and u_R denote the right-handed quark singlets and ϕ denotes the Higgs doublet. We also defined the field $\tilde{\phi}$ by

$$\tilde{\phi} = \epsilon\phi^* = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \phi^0 \\ \phi^+ \end{pmatrix}^* = \begin{pmatrix} -\phi^- \\ \phi^{0*} \end{pmatrix}, \tag{2}$$

and Y^u and Y^d denote the Yukawa coupling matrices. Under CP conjugation, the Yukawa couplings transform as

$$\mathcal{CP}\,Y^{u/d}\,\mathcal{CP}^{-1} = (Y^{u/d})^*, \tag{3}$$

such that imaginary entries in $Y^{u/d}$ potentially constitute CP violation. Spontaneous breakdown of the $SU(2)_L$ symmetry gives then rise to the SM quark masses via

$$\begin{aligned} & Y^u_{ij}\,\bar{u}^i_L\,u^j_R\,\left\langle\phi^0\right\rangle + Y^d_{ij}\,\bar{d}^i_L\,d^j_R\,\left\langle\phi^0\right\rangle + h.c. \\ =\ & \bar{u}_L\,m_u\,u_R + \bar{d}_L\,m_d\,d_R + h.c. \end{aligned} \tag{4}$$

However, not all entries in the Yukawa matrices are observable. The Yukawa couplings are the only terms in the SM Lagrangian that are sensitive to global $SU(3)_R$ flavor transformations. This leads to the conclusion that physical observables can only depend on the combinations $m_u m_u^\dagger$ and $m_d m_d^\dagger$. In addition, there are six global phases in the left-handed quark sector that are unobservable in the SM.

In Ref. [2] it was shown that in perturbation theory the first CP odd combination of the Yukawa couplings that is invariant under these transformations is the so-called Jarlskog determinant

$$\delta_{CP} = \text{Im Det}\left[\frac{m_u m_u^\dagger}{v^2}, \frac{m_d m_d^\dagger}{v^2}\right] = J \prod_{i<j} \frac{\tilde{m}^2_{u,i} - \tilde{m}^2_{u,j}}{v^2} \prod_{i<j} \frac{\tilde{m}^2_{d,i} - \tilde{m}^2_{d,j}}{v^2} \simeq 10^{-19}, \tag{5}$$

where $\tilde{m}^2_{u/d}$ denote the diagonalized mass matrices according to

$$m_d m_d^\dagger = D\tilde{m}_d^2 D^\dagger, \quad m_u m_u^\dagger = U\tilde{m}_u^2 U^\dagger. \tag{6}$$

The identity in Eq. (5) results then from the relation

$$\text{Im}\left[C_{ab}C^\dagger_{bc}C_{cd}C^\dagger_{da}\right] = J\sum_{e,f}\epsilon_{ace}\epsilon_{bdf}, \quad C = U^\dagger D \tag{7}$$

(summation over indices is only performed as explicitly shown) with the Jarlskog invariant J given in terms of the standard parametrization of the CKM matrix C as [2, 3]

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta) = (3.0 \pm 0.3) \times 10^{-5}. \tag{8}$$

The Jarlskog determinant in Eq. (5) reflects the fact that CP violation is absent if any two up-type masses or any two down-type masses are equal. This is required since in this case there is an additional global flavor symmetry that can be used to remove all complex phases from the Yukawa matrices (in the SM case of three quark families).

However, the above argument is based on the assumption that the observable under consideration is perturbative in the Yukawa couplings. For example, CP violation is much larger in the neutral Kaon system than indicated by the Jarlskog determinant. If CP violation in the mixing properties and decay rates of neutral Kaons are considered, the CP-violating effects are suppressed by the Jarlskog invariant J, but not by the Jarlskog determinant δ_{CP}. Experimentally one finds the value [3]

$$\frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle} \approx \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle} \approx 2.2 \times 10^{-3}, \tag{9}$$

which is many orders of magnitude larger than the Jarlskog determinant. This is due to the fact that the initial and final states in the calculation of the decay rates have a well defined quark content and Kaons are distinct from other mesons. If e.g. the strange and bottom quarks would be degenerate in mass, the Kaon would be indistinguishable from the B-mesons and the CP violation in meson decays would be non-observable. However, the quark masses are not degenerate, and the CP violation in the Kaon system is not suppressed by differences in Yukawa couplings as they appear in Eq. (5), but rather depends on ratios of Yukawa couplings and not on the small Yukawa couplings themselves. In this sense, CP violation in the Kaon system is a non-perturbative effect in the quark masses and hence does not need to be suppressed by the Jarlskog determinant [4, 5].

In cosmology, the main interest in CP violation originates from baryogenesis. Sakharov pointed out [6] that CP violation is a prerequisite for any dynamical generation of the observed baryon asymmetry. A baryogenesis mechanism that is based on the SM would be most compelling [7, 8], but this requires that the Jarlskog determinant as an upper bound on CP violation be evaded. Even though non-perturbative effects are obviously present in the QCD sector of the SM, it is not expected that CP violation from the CKM matrix would

play any role in the early Universe, since a viable baryogenesis mechanism can only operate at temperatures higher than the electroweak scale when the sphaleron process provides the needed baryon number violation. On the other hand, at temperatures of the electroweak scale, the quark masses are (besides the top mass) much smaller than the relevant energy scale and hence can be treated perturbatively. It has been argued that in this case, the CP violation might be only suppressed by the temperature rather than by the Higgs vev as given in Eq. (5), but nevertheless this would be insufficient to be significant in a baryogenesis mechanism unless coherent scattering at a first order phase transition bubble wall and a very distinctive behaviour of the various quarks is assumed [4, 9]. This created a controversial discussion [10].

In principle, there are several possibilities to avoid this dilemma and to obtain a significant source of CP violation in the SM as required by baryogenesis. The first option is to consider other rephasing invariants besides the one in Eq. (5). For example, during a first-order phase transition, the Higgs vev changes and hence makes it possible to construct rephasing invariants that do not only contain the masses but also their derivatives that are non-vanishing during the phase transition [11]. However, in the SM these two quantities are proportional to each other, such that no significant enhancement can be obtained. The second possibility is to consider finite temperature effects that in general lead to a break-down of perturbation theory in the infrared. This way, CP violation might be enhanced by several orders of magnitude as demonstrated in Ref. [11], but baryogenesis based on this effect is still implausible.

Finally, CP violation can be considered in the context of effective actions. Consider the SM at low energies with gauge fields that are weak compared to the energy scale of the quark masses. If the fermionic degrees of freedom are integrated out, a purely bosonic theory describes the physics at low energies. In this case, the CP violation in the quark sector will eventually give rise to higher dimensional operators as first proposed in Ref. [12]. In the present work we will demonstrate that, different from the leading order case [12], in the next-to-leading order of the gradient expansion, the effective action indeed contains CP violation that exceeds the perturbative bound given in Eq. (5). Main motivation for this approach is the scenario of cold electroweak baryogenesis [12, 13, 14] that specifically utilizes lattice simulations of the bosonic sector of the SM with higher dimensional operators that violate the CP symmetry.

The approach is based on the determination of the covariant current using the worldline method as presented in Ref. [15]. An alternative and more direct method was recently proposed in Ref. [16]. Even though this direct method nicely avoids the matching of the action to the current, the method we use still exhibits some computational advantages. In particular, the worldline method does not involve momentum integrations, avoids the handling of the γ matrix algebra, and is easily implemented with a computer algebra program.

The paper is organized as follows: In Sec. II we review the effective action in leading order of the gradient expansion. In Sec. III the next-to-leading order effective action is discussed before we conclude in Sec. IV. In Appendix A we comment on some aspects of the explicit form of the effective action at next-to-leading order.

II. THE EFFECTIVE ACTION AT LEADING ORDER

In this section, we present the leading order of the effective action as first presented in Ref. [17] and also derived in Ref. [15] using the worldline method [18, 19, 20]. Besides, we discuss the absence of CP violation at this order following Ref. [12].

Consider the Euclidean Dirac operator

$$\mathcal{O} \equiv \not{p} - i\Phi(x) - \gamma_5\Pi(x) - \not{A}(x) - \gamma_5\not{B}(x), \tag{10}$$

where the external fields have a general internal group structure, e.g. a flavor or gauge matrix structure. We are interested in the imaginary part of the one-loop effective action that contains the CP-violating contributions to the action

$$W^- = \arg\left(\mathrm{Det}\left[\mathcal{O}\right]\right). \tag{11}$$

As shown elegantly in Ref. [21], the imaginary part of the action can be reformulated in terms of variables that have a well-defined behavior under chiral transformations, namely

$$\mathcal{A}_\mu = \begin{pmatrix} A^L_\mu & 0 \\ 0 & A^R_\mu \end{pmatrix} = \begin{pmatrix} A_\mu + B_\mu & 0 \\ 0 & A_\mu - B_\mu \end{pmatrix}, \tag{12}$$

$$\mathcal{H} = \begin{pmatrix} 0 & i\,H \\ -i\,H^\dagger & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\,\Phi + \Pi \\ -i\,\Phi + \Pi & 0 \end{pmatrix}. \tag{13}$$

The fermions under consideration are the quarks of the SM, such that the gauge fields belong to the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group. Since the color interactions are not essential

for CP violation, we suppress any $SU(3)$ indices. Besides a gauge index, the fields also carry a flavor index. In particular, the scalar/pseudo-scalar background field is of the form

$$H = \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & -\phi^{0*} \end{pmatrix} \begin{pmatrix} Y_u & 0 \\ 0 & -Y_d \end{pmatrix}, \tag{14}$$

where $Y_{u/d}$ denote the SM Yukawa coupling matrices.

Since the effective action is gauge invariant, we still have the freedom to simplify the action by a certain choice of gauge. As detailed in Ref. [12] a convenient choice is the unitary gauge, in which the Higgs field is of the form

$$H = \phi^0(x_\mu) \begin{pmatrix} Y_u & 0 \\ 0 & Y_d \end{pmatrix} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}. \tag{15}$$

In addition, we perform a basis transformation that diagonalizes the mass terms m_u and m_d. This transformation is not compatible with $SU(2)_L$ gauge invariance, such that the gauge invariance in the resulting expression is realized non-linearly. In this basis, the $SU(2)_L$ gauge field strength is then of the following form in flavor space

$$\mathcal{F}_L = \begin{pmatrix} F_0 & F^+ C \\ C^\dagger F^- & -F_0 \end{pmatrix}, \tag{16}$$

where C denotes the CKM matrix as defined in Eq. (7).

The effective action is most compactly presented in the labeled operator notation that was introduced in Ref. [17], and used also in Ref. [15]. In this notation, mass matrices obtain an additional subscript that indicates the position of the mass matrix in a subsequent product of operators. For example, using this notation we write

$$m_1 m_2^3 m_3^2\, \mathcal{D}_\mu \mathcal{H}\, \mathcal{D}_\nu \mathcal{H} = m\, \mathcal{D}_\mu \mathcal{H}\, m^3\, \mathcal{D}_\nu \mathcal{H}\, m^2. \tag{17}$$

A detailed definition and applications of this notation can be found in Ref. [17] and we refer the reader to this work. Using this notation, the chiral invariant part of the leading order contribution in the gradient expansion has to be of the form

$$\begin{aligned} W^-_{lo} &= \epsilon^{\mu\nu\lambda\sigma} \big\langle iN(m_1, m_2, m_3)\, \mathcal{D}_\mu \mathcal{H}\, \mathcal{D}_\nu \mathcal{H}\, \mathcal{F}_{\lambda\sigma} \\ &\quad + N(m_1, m_2, m_3, m_4)\, \mathcal{D}_\mu \mathcal{H}\, \mathcal{D}_\nu \mathcal{H}\, \mathcal{D}_\lambda \mathcal{H}\, \mathcal{D}_\sigma \mathcal{H} \big\rangle, \end{aligned} \tag{18}$$

with some functions $N(m_1, m_2, m_3)$ and $N(m_1, m_2, m_3, m_4)$.

In order to contribute to CP violation, an expression has to contain at least four CKM matrices. In this case, the arguments that lead to the relation in Eq. (7) can be used to extract the CP-violating parts. Applying these considerations to the expression in Eq. (18) implies that

- The term proportional to $(\mathcal{DH})^2\,\mathcal{F}$ does not contribute since it contains at most three CKM matrices.

- The term proportional to $(\mathcal{DH})^4$ contains four CKM matrices. In this case all four operators have to be left-handed and charged, i.e. it is proportional to

$$(\mathcal{DH}^2)^+_L(\mathcal{DH}^2)^-_L(\mathcal{DH}^2)^+_L(\mathcal{DH}^2)^-_L \\ \propto (m_2^2-m_1^2)(m_3^2-m_2^2)(m_4^2-m_3^2)(m_1^2+m_4^2)A^+_L A^-_L A^+_L A^-_L, \tag{19}$$

  where the subscripts and superscripts $L,+,-$ denote which parts have been projected out in terms of transformation properties under chiral and $U(1)_{\rm em}$ transformations.

- The contraction of the Lorentz indices of this term with the Levi-Civita ϵ tensor vanishes.

We conclude, in agreement with [12, 15], that the leading order of the effective action in the gradient expansion does not contain any CP violation in the SM. However, in next-to-leading order, the action will contain terms like $(\mathcal{DH})^2\,\mathcal{F}^2$ that do not necessarily vanish after contraction with the Levi-Civita tensor.

III. THE EFFECTIVE ACTION AT NEXT-TO-LEADING ORDER

In this section we discuss some general properties of the effective action in next-to-leading order. First, notice that if written in terms of the gauge field A_μ and the field strength $F_{\mu\nu}$, the coefficient of the effective action has negative mass dimension. Hence, in the limit of vanishing masses, the effective action becomes infinite. This is not surprising, since the gradient expansion assumes

$$A_\mu \ll m, \quad F_{\mu\nu} \ll m^2. \tag{20}$$

This leads to the question what is the range of applicability of our result. In order to discuss this question, we analyze the CP-violating part of a specific term in the effective action.

Consider a term of the form

$$R(m_1, m_2, m_3, m_4)\mathcal{D}_\alpha\mathcal{H}\,\mathcal{D}_\alpha\mathcal{H}\,\mathcal{F}_{\mu\nu}\,\mathcal{F}_{\lambda\sigma}. \tag{21}$$

This could in principle contain CP violation if all appearing gauge fields are left-handed and charged after symmetry breaking. This yields the contributions

$$\begin{aligned}&\bar{R}(m_1^d, m_2^u, m_3^d, m_4^u)\, A_\alpha^+\, A_\alpha^-\, F_{\mu\nu}^+\, F_{\lambda\sigma}^- \\ &\qquad + \bar{R}(m_1^u, m_2^d, m_3^u, m_4^d)\, A_\alpha^-\, A_\alpha^+\, F_{\mu\nu}^-\, F_{\lambda\sigma}^+,\end{aligned} \tag{22}$$

where we used the symmetrization

$$\bar{R}(m_1, m_2, m_3, m_4) = \frac{1}{16}\sum_{n_i \in \pm m_i} R(n_1, n_2, n_3, n_4)(n_2 - n_1)(n_3 - n_2). \tag{23}$$

The symmetrization ensures that all appearing gauge fields are left-handed. Changing to the mass eigenbasis and using Eq. (7) this can be recast as

$$C_1\, A_\alpha^+\, A_\alpha^-\, F_{\mu\nu}^+\, F_{\lambda\sigma}^- + C_2\, A_\alpha^-\, A_\alpha^+\, F_{\mu\nu}^-\, F_{\lambda\sigma}^+, \tag{24}$$

where we use the definitions

$$C_1 = J \sum_{i,k,m \in \mathrm{up}}\ \sum_{j,l,n \in \mathrm{down}} \epsilon_{ikm}\epsilon_{jln}\bar{R}(\tilde{m}_k^d, \tilde{m}_l^u, \tilde{m}_m^d, \tilde{m}_n^u), \tag{25}$$

$$C_2 = -J \sum_{i,k,m \in \mathrm{up}}\ \sum_{j,l,n \in \mathrm{down}} \epsilon_{ikm}\epsilon_{jln}\bar{R}(\tilde{m}_l^u, \tilde{m}_k^d, \tilde{m}_n^u, \tilde{m}_m^d). \tag{26}$$

The subscript indicates hereby the quark flavor, up $= \{u, c, t\}$ and down $= \{d, s, b\}$.

Notice that this expression vanishes if two up-type masses or two down-type masses coincide as required. However, the coefficient can be much larger than the Jarlskog determinant stated in Eq. (5) even in units of the light quark masses $\tilde{m}_{u/d}^{-2}$. The largest contribution results typically from the contribution involving only the four lightest quarks.

Let us come back to the question of the range of applicability of the gradient expansion. In principle, one would expect that the largest contributions be proportional to $\tilde{m}_{u/d}^{-2}$ or even larger, e.g. $\tilde{m}_{c/b}^2\tilde{m}_{u/d}^{-4}$. In this case, the mass scale that indicates the breakdown of the gradient expansion in Eq. (20) would be given by the lightest quarks invalidating the gradient expansion already for very weak external fields. Besides, there might be one more obstacle, namely the physical infrared divergences of the light quarks. The operator under consideration describes a scattering process that is indistinguishable from the same process

including a soft quark/anti-quark pair. Hence, the amplitudes can contain contributions that scale as $\log \tilde{m}_u^2$ or $\log \tilde{m}_d^2$ in the massless limit. This would require that the corresponding operators with soft quarks in the initial/final states be taken into account .

Fortunately, it turns out that all appearing CP-violating contributions are finite in the limit of vanishing up/down quark masses and there are only terms that scale as $\mathcal{O}\left(\tilde{m}_c^{-2}, \tilde{m}_b^{-2}, \tilde{m}_t^{-2}\right)$. We hence expect that the range of validity in Eq. (20) is at least given by the scale of the charm quark mass.

In fact, the range of applicability can be even larger according to the following argument. For simplification, imagine that there is a common energy scale for the gradient expansion

$$A_\mu \sim \partial_\mu \sim E, \quad F_{\mu\nu} \sim \partial_\mu^2 \sim E^2. \tag{27}$$

In the limit of weak fields $E \ll \tilde{m}_c$ we obtain the estimate for CP violation in the effective action

$$W^- \propto J\, \tilde{m}_c^{-2} E^6, \tag{28}$$

while in the case of a strong background, $E \gg \tilde{m}_t$, the effective action could be expanded in the quark masses. In this case, following the argument by Jarlskog, one obtains on dimensional grounds an estimate for CP violation similar to the Jarlskog determinant, namely

$$W^- \propto J\, \tilde{m}_t^4 \tilde{m}_b^4 \tilde{m}_c^2 \tilde{m}_s^2 E^{-8}. \tag{29}$$

Comparison of these two limits indicates that the transition region is given for energies

$$E \sim (m_t^4 \tilde{m}_b^4 \tilde{m}_c^4 \tilde{m}_s^2)^{1/14} \simeq 5.0 \text{ GeV}. \tag{30}$$

and below this value the effective action presented here should indicate the correct order of magnitude of CP violation in the bosonic sector of the SM.

Using the method developed in Ref. [15] we calculated the effective action explicitly. The specific form of the coefficient functions is too large to be presented here, but they are available as computer files [22]. Appendix A contains some more general comments on the action and its coefficient functions.

Interestingly, almost all the contributions cancel amongst themselves, and there is only one contribution to the CP-violating part of the effective action, namely

$$\frac{1}{8(4\pi)^2}\frac{3}{16}\frac{J\,\kappa^{CP}}{\tilde{m}_c^2}\epsilon^{\mu\nu\lambda\sigma}\int d^4x \Big(Z_\mu W^+_{\nu\lambda} W^-_\alpha \left(W^+_\sigma W^-_\alpha + W^+_\alpha W^-_\sigma\right) + c.c. \Big) \tag{31}$$

with J given by Eq. (8) and

$$\kappa^{CP} \approx 9.87. \tag{32}$$

Finally, notice that the action can always be rewritten in $SU(2)_L$ gauge invariant quantities. For example, the charged gauge fields can be rewritten as

$$W^+_{\mu\nu} = \frac{\phi^\dagger W_{\mu\nu}\tilde{\phi}}{\phi^\dagger\phi}, \quad W^-_{\mu\nu} = \frac{\tilde{\phi}^\dagger W_{\mu\nu}\phi}{\phi^\dagger\phi}, \quad W^+_\mu = \frac{\phi^\dagger \mathcal{D}_\mu\tilde{\phi}}{\phi^\dagger\phi}, \quad W^-_\mu = \frac{\tilde{\phi}^\dagger \mathcal{D}_\mu\phi}{\phi^\dagger\phi}, \tag{33}$$

and similarly for the uncharged quantities

$$Z_\mu = W^3_\mu - B_\mu = \frac{\phi^\dagger \mathcal{D}_\mu\phi - \tilde{\phi}^\dagger \mathcal{D}_\mu\tilde{\phi}}{2\phi^\dagger\phi}, \quad h^{-1}\partial_\mu h = \frac{\phi^\dagger \mathcal{D}_\mu\phi + \tilde{\phi}^\dagger \mathcal{D}_\mu\tilde{\phi}}{2\phi^\dagger\phi}, \tag{34}$$

and

$$W^3_{\mu\nu} = \frac{\phi^\dagger W_{\mu\nu}\phi}{\phi^\dagger\phi}. \tag{35}$$

IV. CONCLUSIONS

We calculated the CP-violating contributions to the effective action in the bosonized Standard Model in next-to-leading order in the gradient expansion. Surprisingly after some cancelations only one term remained, given in Eq. (31), our main result. We argued that the resulting action should be valid for bosonic fields whose energy scale does not exceed much the charm mass. This observation is based on the fact that the action after IR regularization remains finite in the limit of vanishing up and down quark masses. We find that the coefficients of the resulting dimension-six operators are suppressed by the charm mass and the Jarlskog invariant J but are many orders larger than the Jarlskog determinant δ_{CP}. It will be interesting to see the results for cold electroweak baryogenesis following the lines of [14]. In principle the temperature enters as an additional mass scale into the calculation. It should be possible to derive an expansion of the effective action that is valid for external fields whose energy scale exceeds the charm mass but not the temperature and that is non-perturbative in the quark masses. This issue is under further study.

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APPENDIX A: EFFECTIVE ACTION AND CP-VIOLATING CONTRIBUTIONS

The imaginary part of the effective action in four dimensions in next-to-leading order in a gradient expansion takes the form

$$\begin{aligned}
W^-_{nlo} \;=\; \epsilon^{\mu\nu\lambda\sigma}\Big\langle &+\frac{1}{4}Q^{(1)}_{123}\mathcal{D}_\alpha\mathcal{F}_{\mu\nu}\mathcal{F}_{\lambda\sigma}\mathcal{D}_\alpha\mathcal{H}+\frac{1}{4}Q^{(2)}_{123}\mathcal{D}_\alpha\mathcal{F}_{\mu\nu}\mathcal{F}_{\lambda\alpha}\mathcal{D}_\sigma\mathcal{H}\\
&+\frac{1}{4}Q^{(4)}_{123}\mathcal{D}_\alpha\mathcal{F}_{\mu\alpha}\mathcal{F}_{\nu\lambda}\mathcal{D}_\sigma\mathcal{H}+\frac{i}{2}Q^{(5)}_{123}\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{H}\mathcal{F}_{\nu\lambda}\mathcal{D}_\sigma\mathcal{H}\\
&+\frac{i}{2}R^{(6)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}+\frac{i}{2}R^{(7)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\alpha\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\\
&+\frac{1}{4}R^{(9)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{F}_{\lambda\alpha}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}+\frac{1}{4}R^{(10)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{D}_\lambda\mathcal{H}\mathcal{F}_{\sigma\alpha}\mathcal{D}_\alpha\mathcal{H}\\
&+\frac{i}{2}R^{(12)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{D}_\alpha\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}+\frac{i}{2}R^{(13)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\alpha\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\\
&+\frac{1}{4}R^{(14)}_{1234}\mathcal{F}_{\mu\nu}\mathcal{D}_\alpha\mathcal{H}\mathcal{F}_{\lambda\sigma}\mathcal{D}_\alpha\mathcal{H}+\frac{1}{4}R^{(15)}_{1234}\mathcal{F}_{\mu\alpha}\mathcal{F}_{\nu\alpha}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\\
&+\frac{i}{2}S^{(1)}_{12345}\mathcal{F}_{\mu\nu}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\alpha\mathcal{H}+\frac{i}{2}S^{(2)}_{12345}\mathcal{F}_{\mu\nu}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\\
&+\frac{i}{2}S^{(3)}_{12345}\mathcal{F}_{\mu\nu}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\sigma\mathcal{H}+\frac{i}{2}S^{(4)}_{12345}\mathcal{F}_{\mu\nu}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\\
&+S^{(7)}_{12345}\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{H}\mathcal{D}_\nu\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}+S^{(8)}_{12345}\mathcal{D}_\alpha\mathcal{D}_\mu\mathcal{H}\mathcal{D}_\nu\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\\
&+T^{(1)}_{123456}\mathcal{D}_\mu\mathcal{H}\mathcal{D}_\nu\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\alpha\mathcal{H}+T^{(2)}_{123456}\mathcal{D}_\mu\mathcal{H}\mathcal{D}_\nu\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\\
&+T^{(3)}_{123456}\mathcal{D}_\mu\mathcal{H}\mathcal{D}_\nu\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\mathcal{D}_\sigma\mathcal{H}\mathcal{D}_\alpha\mathcal{H}\Big\rangle+h.c. \qquad \text{(A1)}
\end{aligned}$$

Due to the trace, partial integration and other possible manipulations, the expression for W^-_{nlo} in Eq. (A1) is not unique. Through a judicious set of such transformations, W^-_{nlo} was brought into a simpler form than the one obtained originally from the matching procedure, into one which is finite term by term at all coincidence limits. The superscripts of the functions distinguish hereby between different coefficient functions with the same number of arguments. The superscripts are not consecutively numbered what is reminiscent of the fact that we obtained this action by removing some contributions of a more general ansatz.

The explicit functions are not shown here for space considerations, but in order to give

the reader an impression of their form, we present the simplest function that is given by

$$\begin{aligned}
Q^{(2)}_{123} =& \frac{8}{(3(m_1^2-m_2^2)^2(m_1+m_2)(m_2^2-m_3^2)^2(m_1+m_3)(m_2+m_3))}\times \\
&\left(m_1^4\left(m_2^2-m_2m_3+m_3^2\right)\left(m_2^2+4m_2m_3+m_3^2\right)+m_2^4m_3\left(2m_2^3-5m_2^2m_3+m_3^3\right)\right. \\
&+m_1^3m_2m_3(m_2+m_3)\left(3m_2^2-2m_2m_3+3m_3^2\right) \\
&+m_1m_2^3(m_2+m_3)\left(2m_2^3-9m_2^2m_3+3m_3^3\right) \\
&\left.+m_1^2m_2^2\left(-5m_2^4-9m_2^3m_3+11m_2^2m_3^2+m_2m_3^3-2m_3^4\right)\right) \\
&+\frac{8m_1^3\left(m_1^4+m_1^3m_2-3m_1^2m_2^2+3m_1m_2^3+6m_2^3m_3\right)\log\left[\frac{m_1^2}{m_2^2}\right]}{3(m_1^2-m_2^2)^3(m_1+m_2)(m_1^2-m_3^2)(m_1+m_3)} \\
&-\frac{8m_3^3\left(6m_1m_2^3+m_3\left(3m_2^3-3m_2^2m_3+m_2m_3^2+m_3^3\right)\right)\log\left[\frac{m_2^2}{m_3^2}\right]}{3(m_1^2-m_3^2)(m_1+m_3)(m_2^2-m_3^2)^3(m_2+m_3)}.
\end{aligned} \tag{A2}$$

All the other functions, while increasing in complexity as the number of arguments increases, are of this form: rational functions of the masses, eventually multiplied by logarithms of mass ratios. In particular, all functions are homogeneous in their arguments for dimensional reasons

$$Q(a\,m_1,a\,m_2,a\,m_3)=\frac{1}{a^2}Q(m_1,m_2,m_3). \tag{A3}$$

The Q functions, lacking sufficient CKM matrices, cannot contribute CP-violating terms. Therefore the CP-violating terms can only appear from the R, S and T functions. As mentioned in the main text, almost all the expressions cancel. Essentially just one of the contributions coming from $R^{(12)}$, $R^{(13)}$ and their conjugates survive, see Eq. (31). In calculating the coefficient in Eq. (32) we used the full analytic functions, but the final result is too big to present it here. However, in the limit where $\tilde{m}_u \to \tilde{m}_d \to 0$ and $\tilde{m}_b \to \tilde{m}_c$ the result is simpler, and differs by around 1% from the one given in Eq. (32). In this limit the

contribution takes the following form

$$
\begin{aligned}
\frac{\kappa^{CP}}{\tilde{m}_c^2} \approx\; & \frac{32}{9\tilde{m}_c^2\left(\tilde{m}_c^2-\tilde{m}_s^2\right)^3\left(\tilde{m}_c^2-\tilde{m}_t^2\right)^3\left(\tilde{m}_s^2-\tilde{m}_t^2\right)^2}\times \\
& \Big(\tilde{m}_s^6\tilde{m}_t^6\left(\tilde{m}_s^2-\tilde{m}_t^2\right)^2+3\tilde{m}_c^{14}\left(\tilde{m}_s^2+\tilde{m}_t^2\right) \\
& -5\tilde{m}_c^2\tilde{m}_s^4\tilde{m}_t^4\left(\tilde{m}_s^2-\tilde{m}_t^2\right)^2\left(\tilde{m}_s^2+\tilde{m}_t^2\right)-12\tilde{m}_c^{12}\left(\tilde{m}_s^4+\tilde{m}_t^4\right) \\
& +\tilde{m}_c^4\tilde{m}_s^2\tilde{m}_t^2\left(\tilde{m}_s^2-\tilde{m}_t^2\right)^2\left(13\tilde{m}_s^4+28\tilde{m}_s^2\tilde{m}_t^2+13\tilde{m}_t^4\right)+18\tilde{m}_c^{10}\left(\tilde{m}_s^6+\tilde{m}_t^6\right) \\
& +\tilde{m}_c^8\left(-12\tilde{m}_s^8+37\tilde{m}_s^6\tilde{m}_t^2-74\tilde{m}_s^4\tilde{m}_t^4+37\tilde{m}_s^2\tilde{m}_t^6-12\tilde{m}_t^8\right) \\
& +\tilde{m}_c^6\left(3\tilde{m}_s^{10}-41\tilde{m}_s^8\tilde{m}_t^2+41\tilde{m}_s^6\tilde{m}_t^4+41\tilde{m}_s^4\tilde{m}_t^6-41\tilde{m}_s^2\tilde{m}_t^8+3\tilde{m}_t^{10}\right)\Big) \\
& -\frac{64\,\tilde{m}_c^4\tilde{m}_s^2\tilde{m}_t^2\left(\tilde{m}_c^2-\tilde{m}_t^2\right)\left(\tilde{m}_c^2-3\tilde{m}_s^2+2\tilde{m}_t^2\right)\log\left[\frac{\tilde{m}_s^2}{\tilde{m}_c^2}\right]}{3\left(\tilde{m}_c^2-\tilde{m}_s^2\right)^4\left(\tilde{m}_s^2-\tilde{m}_t^2\right)^3} \\
& +\frac{64\,\tilde{m}_c^4\tilde{m}_s^2\left(\tilde{m}_c^2-\tilde{m}_s^2\right)\tilde{m}_t^2\left(\tilde{m}_c^2+2\tilde{m}_s^2-3\tilde{m}_t^2\right)\log\left[\frac{\tilde{m}_t^2}{\tilde{m}_c^2}\right]}{3\left(\tilde{m}_c^2-\tilde{m}_t^2\right)^4\left(\tilde{m}_s^2-\tilde{m}_t^2\right)^3}.
\end{aligned}
\tag{A4}
$$

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